described, and others of similar nature. The word wave-meter may probably be preferred in practice, but, if a special term is desired, the author suggests, with diffidence, the name *Cymometer* or *Kymometer* (from κῦμα, a wave) as applicable to it.*

"The Effects of Momentary Stresses in Metals." By BERTRAM HOPKINSON, M.A., Professor of Mechanism and Applied Mechanics in the University of Cambridge. Communicated by Professor EWING, F.R.S. Received January 31,—Read February 16, 1905.

In 1872 the late Dr. John Hopkinson published an investigation into the effect of a blow delivered by a falling weight on the lower and free end of a wire, the upper end of which is fixed.† It is unnecessary to repeat the mathematical analysis in full, but its main features appear in the following argument:—As soon as the weight strikes the stop at the lower end a wave of extension starts up the wire, and the velocity with which it is propagated is $\sqrt{E/\rho} = a$, where E is Young's modulus, and ρ the density of the wire. At a time t after the weight has struck, so short that its velocity is not appreciably diminished, the lower end of the wire has moved through a distance Vt, where V is the velocity of the weight immediately after striking. That is to say, the wire as a whole is lengthened by an amount Vt. This extension is felt over a distance at from the lower end, that being the distance through which the wave of extension initiated by the blow has travelled. The mean strain in this portion of the wire is therefore V/a, and the remainder of the wire is not extended. The wave now travels up the wire to the fixed end, and when it reaches there a reflected wave of equal amplitude starts down the wire. There results momentarily at the top end of the wire a strain equal to $2V/a$ with a corresponding tension $2EV/a$. This is the maximum tension experienced by any part of the wire until the reflected wave again reaches the lower end.

Each bit of the motion of the weight after striking contributes an element to the wave of extension, which is proportional to the then velocity of the weight. The weight is continually being retarded, and the amplitude of the wave therefore continually diminishes as you go back from its front.

* The writer is indebted to his colleague, Professor A. Platt, for advice on the correct form of these words.

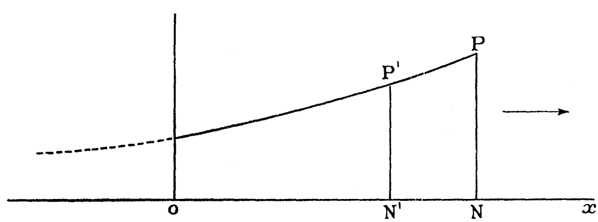
† 'Original Papers,' Hopkinson, vol. 2, p. 316.

In Fig. 1 the abscissæ are distances measured from O, the free end of the wire; the ordinates are the strains, and any one of them, say P′N′, is equal to v/a, where v is the velocity which the weight had when the bit of wave at P′ left it. If t be the time, reckoned from the moment of striking, $\mathrm{ON} = at$, and it is easy to show that

$$\mathrm{P'N'} = \mathrm{PN}e^{-\frac{\mu\,.\,\mathrm{NN'}}{\mathrm{M}}} = \frac{\mathrm{V}}{a}\,e^{-\frac{\mu}{\mathrm{M}}(at-x)},$$

where μ is the mass of the wire per unit length, and M the mass of the weight. The wave of extension represented by the curve and its dotted continuation travels up the wire without change of type to the upper end, where it is reflected, and a similar wave travels down the wire, the effect of which is added to that of the original wave. The strain at any point of the wire, such as N′, is zero till the wave reaches it. The strain then becomes V/a, and gradually diminishes according to the exponential law $e^{-\mu at/\mathrm{M}}$ until the reflected wave reaches N′, when the strain increases by V/a again. Further reflection will occur

FIG. 1.



at the moving weight, but in my experiments this is not considerable, and the maximum strain experienced at any point of the wire, at any rate in the upper half, occurs when the reflected wave reaches it. If x' be the distance of the point from the *upper* end, the total strain due to the up-going and down-coming waves then is $(1 + e^{-2\mu x'/\mathrm{M}})\dfrac{\mathrm{V}}{a}$. The movement in space of any point N′ before the reflected wave reaches it is equal to the area of the curve PNN′P′. For the point in question this is $\dfrac{\mathrm{MV}}{\mu a}(1 - e^{-2\mu x'/\mathrm{M}})$. The strain caused by the blow is added to any initial strain in the wire. When, as is usually the case, the wire is under tension at the moment of the blow, and the tension is released by the blow, the initial strain in the wire is somewhat diminished by the time the wave reaches the top end; superposed upon the extension caused by the blow there is a slight contraction due to the release of the tension at the lower end at the moment of striking. The ultimate

result is that the total increase of length, caused by the blow, of a piece x' at the top end of the wire is

$$\frac{\mathrm{MV}}{\mu a}\left(1-e^{-\frac{2\mu x'}{\mathrm{M}}}\right)-\tfrac{1}{2}\frac{\mathrm{T}}{\mathrm{M}}\left(\frac{2x'}{a}\right)^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where T is the initial tension. In my experiments $2\mu x'/\mathrm{M}$ is small, and its square may be neglected. The expression then becomes

$$2x'\frac{\mathrm{V}}{a}\left(1-\frac{\mu x'}{\mathrm{M}}\right)-\tfrac{1}{2}\frac{\mathrm{T}}{\mathrm{M}}\left(\frac{2x'}{a}\right)^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots \quad (2).$$

The second term is a small correction, but cannot in all cases be neglected. The piece of wire lengthens continuously as the wave passes over it, and begins to contract when the reflected wave arrives at its lower end. The extension then has the value given by expression (1). These results are all to be found in Dr. Hopkinson's papers cited above, or follow at once from the results there given; and so it does not seem necessary to repeat the proofs here.

In the same paper Dr. Hopkinson gave the result of some rough experiments which went to confirm the principal conclusion from this analysis, namely, that the power of a blow to rupture a wire should be measured rather by the velocity with which it is delivered than by its energy or its momentum. It also appeared, as might be expected, from the mathematics, that the wire was most likely to break at the upper end.

In these experiments, made over 30 years ago, the only available means of estimating the momentary stresses produced by the blow was the effect they left upon the wire, *e.g.*, rupture. As the mathematical treatment proceeds upon the assumption that the stress and strain are everywhere and always proportional, it was not to be expected that it could give more than a very general indication of the impulse necessary to rupture the wire. With the appliances now available, however, I think that experiments on these lines are capable of yielding a good deal of information about the effect of stresses applied for a very short time, such as are met with in most cases of shock. The practical importance of such information need not be insisted upon.

I have, therefore, made some experiments of the same kind, but instead of rupturing the wire I have used blows which leave but little permanent extension. I have measured the momentary extension of a few inches at the top of the wire, and compared this with the extension as calculated from theory and given in expression (1) above. If the two agree, and if not much permanent extension is left, it is clear that the theory is correctly applied, and that the stresses in the material may be calculated from it. Moreover, we know that the material must be substantially elastic up to the maximum stress so calculated if applied for the time given by the theory.

The general result that I have obtained is that iron and copper wires may be stressed much beyond the static elastic limit and even beyond their static breaking loads without the proportionality of stresses and strains being substantially departed from, provided that the time during which the stress exceeds the elastic limit is of the order of 1/1000 second or less.

The wire was in each case of No. 10 gauge, and about 30 feet long; it was hung in a vertical chase in a wall, the upper end being firmly fixed in a block of iron, weighing about 20 lbs., the ends of which were built into the wall. This block carried a vertical steel rod, at any point of which could be clamped the contact-making device for measuring the momentary extension. The construction of this sufficiently appears from the figure. The light hard steel point A is fixed to the wire at a certain distance, usually 20 inches, from the upper end. The wire having been drawn taut preparatory to the experiment, the insulated spring S is pushed up by the micrometer screw until contact is made with the point as shown by the deflection of the galvanometer. The spring is then withdrawn by the amount of extension expected; the blow is delivered and the galvanometer shows whether contact between the point and the spring has occurred or not.

By using a sensitive ballistic galvanometer without any resistance in series with it, it was found quite easy to determine the instantaneous extension of 20 inches of wire correct to 1/1000 of an inch, that amount of difference in the position at which the spring is set converting a big throw of the galvanometer into no deflection at all. In a few cases a second point was added with a similar contact spring close to the upper end of the wire, in order that any displacement of the wire relative to its supports might be detected. But I found that if the wire was soft soldered into a bolt about 3 inches long screwed into the block, this precaution was unnecessary.

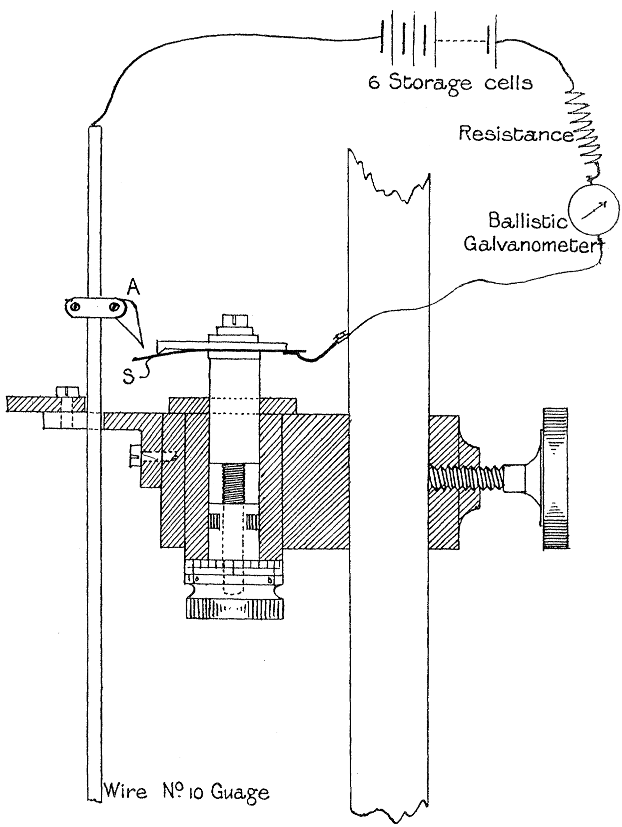
The falling weight was a cylindrical piece of steel weighing 1 lb., with a hole drilled along its axis which was a loose fit on the wire. The wire was kept taut by a spring balance attached to its lower end, the tension in which could be varied from 20 to 200 lbs. The stop struck by the falling weight was a metal sleeve slipped over the wire and soldered to the lower end. It was made as light as possible so that the velocity of the weight should not be much diminished at impact. The velocity at impact was calculated from the height of fall, being taken as $\frac{M}{M+m}\sqrt{2gh}$, where M is the mass of the weight, and m that of the stop.

From time to time the permanent extension on the 20 inches was measured by again pushing the spring into contact with the wire when under steady tension. No analyses were made of the materials, because, at the present stage, all that I desire to do is to compare the

effects in the same material of impulsive and of long continued stress. The nature of the material, however, sufficiently appears from the static tests.

Iron Wire.—This was bought as iron wire, No. 10 gauge. Its diameter was 0·1275 inch. After placing it in the chase it was heated to redness and stretched, to straighten it. The annealing softened it materially, and somewhat unequally in different parts.

Fig. 2.



The following was a set of observations on this wire, which is typical:—

Steady tension 50 lbs.—Height of fall 5 feet.

Contact point 20 inches from the upper end.

One division on micrometer head = 1/2000 inch = 1/40000th part.

Micrometer reads 34·5 with a steady load of 50 lbs. when contact first made on the spring being pushed up.

Micrometer set at 110·5 (76 divisions extension). No contact when weight let fall.

Repeat the blow.—No contact. Repeat.—No contact.

Micrometer with steady load of 50 lbs. now reads 34·5.

Micrometer set at 109·5 (75 divisions extension). No contact.

Altered to 108·5. Contact occurred; the galvanometer spot went half across the scale.

Repeat.—Contact again.

Steady micrometer reading after this 36·0.

Micrometer set at 111 (75 divisions extension). No contact.

Repeat.—No contact. Repeat.—No contact.

Steady reading now 36·0.

Micrometer set at 110 (74 divisions extension). No contact.

Repeat.—Contact occurred.

Steady reading now 37·0.

Hence, the instantaneous extension in this case is 74 micrometer divisions, and the permanent extension produced by 11 blows is about 2·5 divisions.

Then followed a set of 4 blows with a 10-foot fall, and with 20 lbs. tension. They resulted as follows:—

Steady reading 7·0. Set at 110 (103 divisions). Contact.
Steady reading 20·0. Set at 125 (105 divisions). No contact.
Steady reading 29·5. Set at 132 ($102\frac{1}{2}$ divisions). Contact.
Steady reading 42·0. Set at 148 (106 divisions). No contact.
Steady reading 52·5.

Hence, the extension produced by the blow is probably between 103 and 105 divisions, and almost certainly between 102 and 106. Permanent extension produced by 4 blows = 45·5 divisions, or just over 1/1000th part.

Now static tests on this wire showed that a load of 390 lbs. extends it by 1/1000th part. Also μ its mass per foot is 0·0435 lb. Hence

$$a = \sqrt{\frac{\mathrm{E}}{\mu}} = \sqrt{\frac{390{,}000 \times 32}{0{\cdot}0435}} = 17{,}000 \text{ f.s. nearly.}$$

The mass of the stop was 0·04 lb. Hence V, the velocity just after impact, is, with a 5-foot height of fall, 17·2 f.s. Also since $x' = 1{\cdot}66$ feet; $\frac{\mu x'}{\mathrm{M}} = \frac{1{\cdot}66 \times 0{\cdot}0435}{1} = 0{\cdot}07$.

Further, $\mathrm{T} = 50$ lbs. $= 1600$ poundals, and it will be found that $\frac{1}{2}\frac{\mathrm{T}}{\mathrm{M}}\left(\frac{2x'}{a}\right)^2$ is about 0·4 thousandth of an inch, or 0·8 micrometer division. Substituting these figures in the expression (2) above, the extension on the 20 inches, as calculated, is 37·2 thousandths of an inch, or, say, $74\frac{1}{2}$ micrometer divisions. The observed extension is 74 divisions, which is close agreement. This, coupled with the fact

that the permanent extension produced on the 20 inches is negligible, is fairly conclusive evidence that the theory is applicable in this case, and that the material is almost perfectly elastic up to the highest stress as calculated from the theory. The maximum strain at the top of the wire is $2V/a$, or $34 \cdot 4/17$ thousandths. The corresponding tension is about 790 lbs. To this must be added the 50 lbs. steady tension, making a total of 840 lbs. as the maximum stress experienced by any portion of the wire. The mean tension in the top 20 inches, where the extension is greatest, is $\left(\frac{74}{40,000} \times 390\right) + 50$, or 770 lbs.

Now, after the completion of the experiments, the top 20 inches of the wire were cut out and tested statically with a Ewing's extensometer. There was perceptible failure of elasticity at 500 lbs., very marked yielding at 700 lbs. (which produced a permanent extension of nearly 1 per cent.), and at 800 lbs. the wire drew out very rapidly and finally broke.

With a fall of 10 feet and a steady tension (T) of 20 lbs., the calculated extension will be found to be 53 thousandths of an inch, or 106 micrometer divisions. This, again, agrees very well with the observed extension of 104 divisions. In this case the calculated maximum tension at the top end is 1150 lbs., and the mean tension on the 20 inches about 1000 lbs. Of this extension, however, about 11 per cent. is permanent, so that there is some failure of elasticity, and it is improbable that the maximum stress quite reaches the calculated value. It is practically certain, however, that it exceeds the mean stress corresponding to the *elastic* part of the maximum extension in the top 20 inches, viz., about 900 lbs.

Next as regards the time for which these stresses are applied. The strain at the top of the wire is $2\frac{V}{a}e^{-\mu a t/M}$, where t is the time which has elapsed since the wire first arrived there. In the case of the 5-foot fall it will be found that the stress falls from 840 lbs., its initial and maximum value, to 500 lbs., which may be taken as the elastic limit, in about 0·8 thousandth of a second.

These results were fully confirmed by a large number of experiments in which different steady tensions were applied. The general conclusion is that in this material, which has an elastic limit of 40,000 lbs., or 17·8 tons per square inch, and breaks at 28·5 tons, a stress momentarily exceeding 75,000 lbs., or $33\frac{1}{2}$ tons, and exceeding the static elastic limit for a time of the order of 1/1000 second, may be applied without any very great failure of elasticity.*

It may be further noted that a blow from a height of 10 feet,

* The absolute stress is, as usual, calculated on the uncontracted area of the test-piece. The statical breaking stress at the moment of breaking is, of course, greater than this figure, but then the material is hardened by the drawing out.

giving a tension momentarily exceeding 900 lbs., produces a permanent extension of 1/3500th part, or 1/30 of the ultimate extension caused by a steady load of 700 lbs.

Copper Wire.—This was 0·129 inch diameter, and of the kind used in electric light cables. It was set up without preparation of any kind. A load of 220 lbs. stretched it by 1/1000th part, corresponding to $E = 7500$ tons per square inch. The wire weighed 0·0503 lb. per foot. The steady tension was 200 lbs. Mass of falling weight (M) 0·945 lb.; mass of stop, 0·023 lb. Velocity of propagation of waves, a, $= 11{,}800$ f.s.

With a fall of 12·6 inches the additional extension observed on the top 20 inches was 41 micrometer divisions. The calculated extension (formula (2) above) is 43 divisions. The permanent extension produced by 20 blows was about 2 micrometer divisions.

With a fall of 2 feet 6 inches the observed extension was 67 divisions. The calculated extension is $70\frac{1}{2}$ divisions. Ten such blows extend the 20 inches by 13·5 micrometer divisions, or 6·7 thousandths of an inch.

The elasticity is therefore practically perfect up to the stresses caused by a fall of 2 feet 6 inches. The greatest strain at the top end caused by this blow is $2V/a = 2{\cdot}10$ thousandths. The tension due to this is 440 lbs., and the resultant tension, including the initial 200 lbs., is 640 lbs. The mean tension on the top 20 inches is 570 lbs. (calculated from the observed extension of 67 divisions).

Tested statically with the extensometer, this wire showed failure of elasticity at 500 lbs. With a load of 590 lbs. it yielded rapidly and finally broke.

With a fall of 5 feet, and the same initial tension of 200 lbs., the observed extension on the top 20 inches was between 100 and 105 micrometer divisions, as against 103 calculated; of these about 30 divisions were permanent. The calculated maximum tension in this case (including the 200 lbs.) is 890 lbs. But the elasticity here is far from perfect, and the actual stress is probably somewhat less than the calculated value.

The observed extensions are, in the case of the copper wire, about 5 per cent. less than the calculated. I think that this is more than can be accounted for by errors of observation, or by such causes as friction between wire and weight, especially having regard to the much closer agreement in the other wires with which I have experimented. A possible explanation is that in the copper wire the value of Young's modulus for these extremely rapid extensions is 10 per cent. greater than for slowly applied forces. The difference between the adiabatic and isothermal elasticities as calculated from the coefficient of expansion and Young's modulus, is not sufficient to account for the effect, which must be a true time effect if it exists.

The history of the stress in a section of the wire after one of these

blows is rather complicated, and it is difficult to deduce from the results anything more than the general conclusion stated above, that the wire is substantially elastic up to stresses much beyond the static elastic limit, and that the mathematical theory gives correct results. I hope, however, by suitable modifications of the experiment, to simplify the conditions, and obtain by this method more detailed information as to the properties of materials when subjected to shock. It seems to me quite possible that the stress-strain relations for stresses beyond the elastic limit may be much simplified if the stresses are applied for exceedingly short times, because the complication of hardening, due to overstraining, will be to a large extent removed.

"Phosphorescence caused by the Beta and Gamma Rays of Radium." By G. T. BEILBY. Communicated by Professor LARMOR, Sec. R.S. Received January 25,—Read February 9, 1905.

1. Various observers have noticed that barium platino-cyanide, after continued exposure to the rays from radium, becomes brown or red, while the phosphorescence excited by the rays falls off considerably.* The following observations were made with the object of ascertaining the conditions under which this change occurs.

2. A specimen of Merck's barium platino-cyanide was recrystallized and obtained in prisms from 3 to 5 mm. long. The crystals were bright canary yellow and showed a pale blue fluorescence by obliquely reflected light.

3. The radium used was 30 milligrammes of pure bromide contained in a cell with a thin mica cover. This radium is the property of Mr. Frederick Soddy, to whom I am much indebted for granting me its exclusive use for more than three months. All the experiments were made without removing the mica cover of the cell, so that the effects produced were due entirely to the β and γ rays. In comparing the phosphoresence at different stages, black paper was interposed between the cell and the substance so as to cut off the luminous rays from the radium.

4. When yellow crystals of platino-cyanide are left on the mica cover of the radium cell for half an hour, the beginnings of the colour change from yellow to red are distinctly visible. In one hour those surfaces most directly in the path of the rays become strongly reddened. In eight hours the phosphorescence has fallen to a minimum of 8/100 of its original amount, at which it remains, however long the exposure may be continued.

Crystals were exposed to the rays for eight days, but the phosphor-